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Variation of the Electrical Characteristics of an Inhomogeneous Microstrip Line with the Dielectric Constant of the Substrate and with the Geometrical Dimensions

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***Abstract*—Studying systematically the variations of electrical characteristics of microstrip lines with the width w of the line, the thickness h, and the dielectric constant ϵ_r of the substrate, we have obtained a perfect linear variation with ϵ_r. Then using a least squares method, we have been able to give an analytical expression of capacitances usable for $1 \leq \epsilon_r \leq 100$ and $0.04 \leq w/h \leq 10$. The importance of this result is that we can give impedances and phase velocities without any computation.**

It is well known that all the parameters of microstrip lines or couplers can be calculated from the elements of the matrix of capacitances (S) for a purely dielectric substrate [1]. The matrix of the inductances (M) can be deduced from the last one by the relation

$$(M) = \frac{1}{c^2}(S_0)^{-1}$$

where c and S_0 are, respectively, the velocity of light and the matrix (S) in the vacuum.

Up to now, in the case of the quasi-TEM approximation the capacitances can be computed by different numerical methods [2]–[4]. These methods are lengthy and not very easy to use. We have used an accelerated finite differences method [5] which is more convenient (and also universal) to calculate these capacitances, but we have thought that it would be very interesting for practical uses to derive a formula giving directly the capacitances of the microstrip lines and couplers only in terms of the geometrical parameters and the dielectric substrate. Knowing the capacitances, we can obtain impedances, effective dielectric constant, coupling coefficient, etc.

Another interesting point is that such a formula will be able to permit users to define the geometrical dimensions of the lines to obtain a fixed value of the capacitance for a given dielectric substrate.

First, we have studied the single inhomogeneous microstrip line. Fixing w/h, we have calculated the capacitance for several values of ϵ_r between 1 and 100. Plotting the capacitances in terms of ϵ_r, we have obtained a straight line for any value of w/h (Fig. 1). The slope p of these straight lines increases with w/h.

We can write $C = p(\epsilon_r - 1) + C_0$, where p depends only on w/h. After this, we have plotted p in terms of w/h with the purpose of obtaining a polynomial approximation of this function, the graph of which would cross over at the calculated points. We have obtained a curve having an oblique asymptotic direction for the large values of w/h (Fig. 2).

[For the small values of w/h, we have taken $h = 50$ instead of $h = 8$ to improve the precision of the computation (Fig. 3).]

This is physically correct because when w/h becomes very large, the capacitance of the line tends to the one of a perfect plane capacitor

$$C = \epsilon_0\epsilon_r \frac{w}{h} = \epsilon_0 \frac{w}{h}(\epsilon_r - 1) + \frac{\epsilon_0 w}{h}.$$

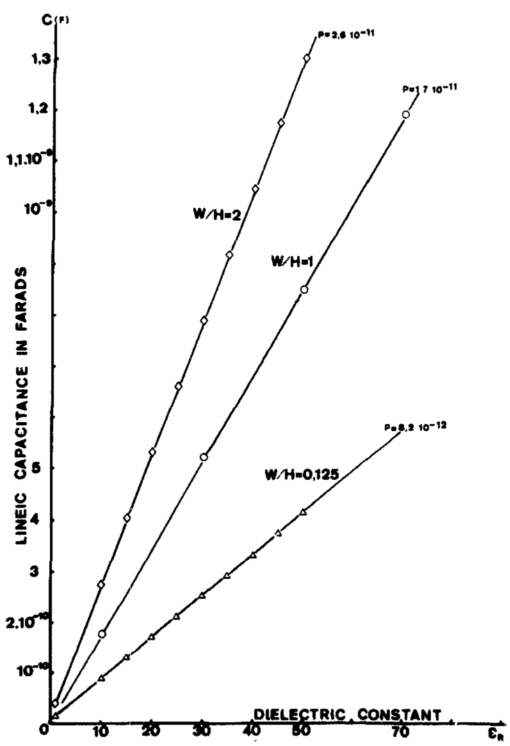

Fig. 1. Variation of the linear capacitance of the line with the dielectric constant of the substrate.

Manuscript received October 4, 1974; revised February 19, 1975.
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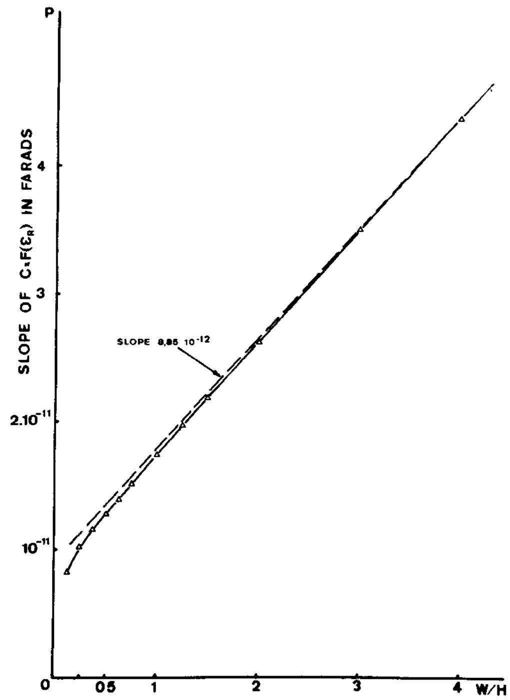

Fig. 2. Slope of the function $c = f(\epsilon_r)$ in terms of w/h.

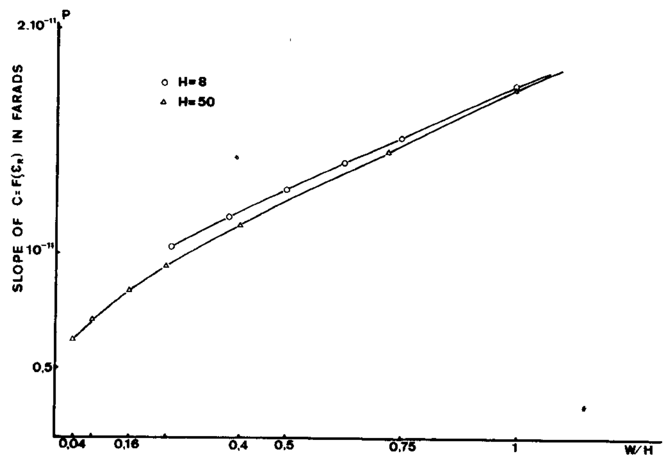

Fig. 3. Slope of the function $c = f(\epsilon_r)$ for small values of w/h.

So if w/h tends to infinity, p becomes equivalent to $\epsilon_0(w/h)$ and C_0 to $\epsilon_0(w/h)$. Then we can give a limited development of p

$$p = \epsilon_0 \frac{w}{h} + a_0 + a\left(\frac{w}{h}\right)^{-1} + \cdots + a_n\left(\frac{w}{h}\right)^{-n}.$$

By a "least squares method," we have calculated the coefficients $a_0, a \cdots a_n$ of this expression.

As a matter of fact, knowing N values of $p(p_i)$ corresponding to N values of $w/h((w/h)_i)$, we can define an expression Φ by

$$\Phi^2 = \sum_{i=1}^{N}\left[\epsilon_0\left(\frac{w}{h}\right)_i + a_0 + a_1\left(\frac{w}{h}\right)_i^{-1} + \cdots + a_n\left(\frac{w}{h}\right)_i^{-n} - p_i\right]^2.$$

This expression is minimum when the values of $a_0, a_1 \cdots a_n$ are such as the values of the development of p are as close as possible to p_i. This is so if all the partial derivatives $[\partial(\Phi^2)]/\partial a_j$, $0 \leq j \leq n$ are equal to zero.

Now we have

$$\frac{\partial(\Phi^2)}{\partial a_j} = \sum_{i=1}^{N} 2\left(\frac{w}{h}\right)_i^{-j}\left[\epsilon_0\left(\frac{w}{h}\right)_i + a_0 + a_1\left(\frac{w}{h}\right)_i^{-1} + \cdots + a_n\left(\frac{w}{h}\right)_i^{-n} - p_i\right]$$

or

$$\frac{\partial(\Phi^2)}{\partial a_j} = 2\sum_{i=1}^{N} \epsilon_0\left(\frac{w}{h}\right)_i^{1-j} + a_0\left(\frac{w}{h}\right)^{-j} + a_1\left(\frac{w}{h}\right)_i^{-1-j} + \cdots + a_n\left(\frac{w}{h}\right)_i^{-n-j} - p_i\left(\frac{w}{h}\right)_i^{-j}.$$

So

$$\frac{\partial(\Phi)^2}{\partial a_j} = 0, \qquad 0 \leq j \leq n$$

if

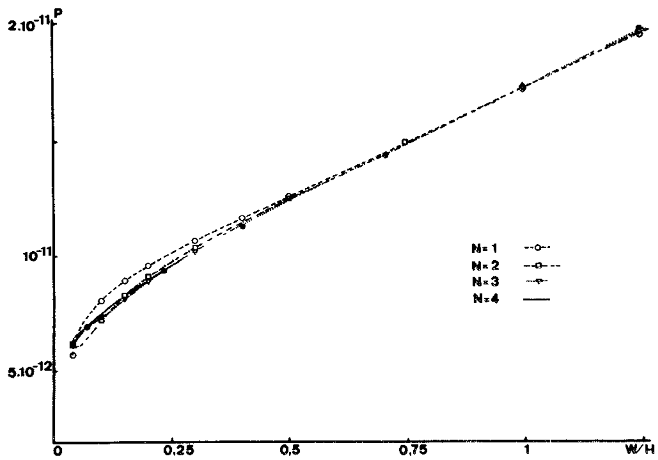

Fig. 4. Polynomial approximation of p for different values of the degree. Small values of w/h. ●—initial values.

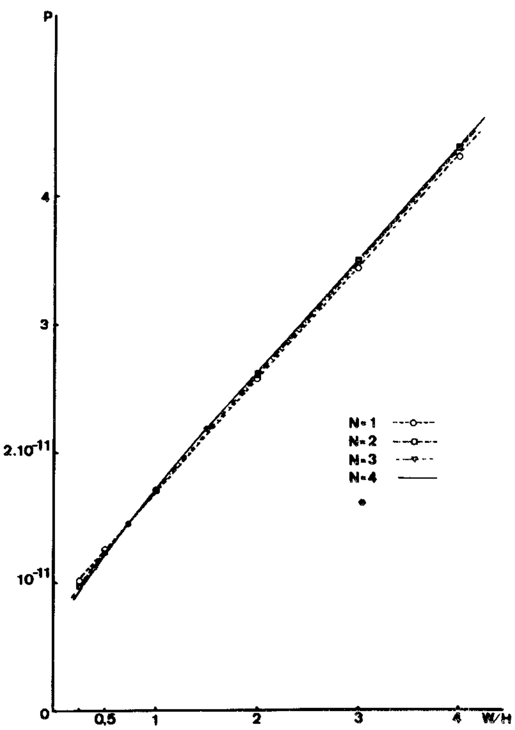

Fig. 5. Polynomial approximation of p for different values of the degree. Great values of w/h. ●—initial values.

$$\epsilon_0 \sum_{i=1}^{N} \left(\frac{w}{h}\right)_i^{-j+1} + a_0 \sum_{i=1}^{N} \left(\frac{w}{h}\right)_i^{-j} + a_1 \sum_{i=1}^{N} \left(\frac{w}{h}\right)_i^{-j-1} + \cdots + a_n \sum_{i=1}^{N} \left(\frac{w}{h}\right)_i^{-j-n} - \sum_{i=1}^{N} p_i \left(\frac{w}{h}\right)_i^{-j} = 0$$

or

$$a_0 \sum_{i=1}^{N} \left(\frac{w}{h}\right)_i^{-j} + a_1 \sum_{i=1}^{N} \left(\frac{w}{h}\right)_i^{-j-1} + \cdots + a_n \sum_{i=1}^{N} \left(\frac{w}{h}\right)_i^{-j-N} = \sum_{i=1}^{N} p_i \left(\frac{w}{h}\right)_i^{-j} - \epsilon_0 \sum_{i=1}^{N} \left(\frac{w}{h}\right)_i^{-j+1}, \qquad 0 \le j \le n.$$

We obtain a linear system of $(n+1)$ equations with $(n+1)$ unknowns. Taking successively $n = 1$ to 9, we have obtained, with $\epsilon_0 = 8.85 \times 10^{-12}$ and p expressed in farads:

$n = 1$ $a_0 = 8.40 \times 10^{-12}$ $a_1 = -1.25 \times 10^{-13}$

$n = 2$ $a_0 = 8.67 \times 10^{-12}$ $a_1 = -3.12 \times 10^{-13}$ $a_2 = 8.00 \times 10^{-15}$

$n = 3$ $a_0 = 8.80 \times 10^{-12}$ $a_1 = -4.75 \times 10^{-13}$ $a_2 = 3.02 \times 10^{-14}$ $a_3 = -3.67 \times 10^{-16}$

$n = 4 \quad a_0 = 8.81 \times 10^{-12} \quad a_1 = -4.92 \times 10^{-13} \quad a_2 = 3.54 \times 10^{-14} \quad a_3 = -1.08 \times 10^{-15} \quad a_4 = 1.05 \times 10^{-17}$

etc.

It is clear that it is not necessary to take a very long development. The practical devices have generally a geometry such as $w/h \geq 1$, so it is not necessary to give a fourth-order development for p because in the third-order one the ratio $|a_3/a_2|$ is very small ($\sim 2 \times 10^{-2}$). But if w/h is very small ($w/h < 0.1$), it would be necessary to take a larger n.

Figs. 4 and 5 give the graphs of the developments of p and the positions of the computed points. We can see on these graphs that a fourth-order development is sufficient to give a good representation of the variation of p with w/h.

Finally, we have the capacitance of a single microstrip line in terms of ϵ_r, w/h, and C_0. C_0 is the capacitance of the line without dielectric substrate. By a conformal mapping method [6], it is possible to calculate this last capacitance only in terms of ϵ_0 and w/h. Then the final result is that we have a formula giving the capacitances for all the possible configurations.

The same result can be obtained for the coupled line. We are currently working on this problem.

These results are very important, because they make very lengthy and expensive computations unnecessary in order to obtain impedance for the microstrip line, coupling coefficient, and adaptation for couplers and also phase displacement and group delay for meander lines.

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Inductance of Nonstraight Conductors Close to a Ground Return Plane

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***Abstract*—Measurement and calculation of the inductance of a nonstraight conductor close to ground return plane are considered. An equivalent circuit model solution is given, and the results are compared to measurements for a corner-type geometry. Much larger changes in inductance as a function of frequency have been observed for the corner-type geometry than for the equivalent straight-conductor geometry. The circuit model can be used to predict the inductance for other configurations.**

Manuscript received August 16, 1974; revised April 4, 1975.

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I. INTRODUCTION

Although strip-type transmission lines are considered to be mostly two-dimensional, they often have corner-type discontinuities. Typical examples are wires in ceramic digital circuit packages [1] or stripline filter applications [2]. Two cases can be distinguished depending on whether ground planes are close to the wires or are remote (or entirely absent). A three-dimensional formulation based on a low-frequency approximation [3] leads to rather accurate inductance values for wires without nearby ground planes. Essentially, the current redistribution with frequency, usually called skin effect, leads to only a small variation in inductance. Two changes take place in inductance for wires close to a ground return plane for the L-shaped geometry shown in Fig. 1. A relatively small inductance variation is associated with the redistribution of current in the strip conductor [4] while, as shown here, a very large change in inductance can be associated with the redistribution of current in the ground plane. For high frequencies, the current will flow under the strip conductor since this provides a low-inductance path. At lower frequencies, current will flow across the ground plane due to the lower resistance and larger inductance associated with this path. The onset of the two changes in inductance depends on the relative conductance and thickness of the strip conductor compared to the ground plane. The model used in [4], where a perfect image is assumed, may be sufficient at microwave frequencies for a strip conductor close to a highly conductive substrate. The ground-plane effect discussed here is more important for time-domain applications where the pulses contain a large spectrum of frequencies, as well as for low-conductance substrates like semiconductor chips.

Another important aspect associated with the L-shaped conductor geometry to be considered is the high-frequency discontinuity inductance which is reported elsewhere [5], [6]. Thus the work presented in this short paper, when used in conjunction with that referenced previously, leads to a characterization of the L-shaped conductor geometry for a wide range of frequencies.

In Section II, a partial-element equivalent circuit (PEEC) model [7] will be presented, while a comparison between the computer model and measurements will be given in Section III.

II. PARTIAL-ELEMENT EQUIVALENT CIRCUIT MODEL

The reader is referred to [7] for details concerning the PEEC model employed. Simple considerations show that the capacitances can be ignored for the physical structure analyzed here over the frequency range of interest. The equivalent circuit is based on the cells into which the structure is divided as shown in Fig. 2. The nodes indicated coincide with the nodes in the circuit model. The equivalent circuit corresponding to the numbered nodes in Fig. 2 is given in Fig. 3, and only the elements in the X direction are labeled. Note that this is only a small portion of the rather complex circuit. The construction of the total circuit should, however, be evident. Coupling exists among all the partial inductances corresponding to current in the same direction, while all inductances with cell currents perpendicular to one another are decoupled. The

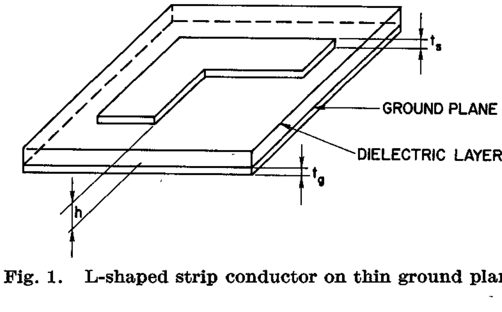

Fig. 1. L-shaped strip conductor on thin ground plane.